УДК 372.8

DIFFERENTIAL EVOLUTION IN SCHOOL MATHEMATICS - A DERIVATIVE-FREE METHOD FOR CALCULATING EXTREME VALUES

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By the help of a derivative-free search method (differential evolution), the parameter estimation for a nonlinear regression function (binary logistic regression) is carried out. Random data material is available: Depending on the volume x (x=0 "very quiet" to x=5 "very loud") the ringing of an alarm clock leads to the fact that one "continues to sleep" (category y=0) or "wakes up and get up"(category y=1). It can happen that a quieter ringing is already heard and a louder ringing is not heard. The example is suitable for school mathematics and shows connections to the theory of probability.
Keywords: *Relationship between school mathematics and probability theory, derivative-free search method, differential evolution, parameter estimation, binary logistic regression, nonlinear regression, method of least squares, vector calculation.*

Preface:

Ministry of Education in Saxony/Germany introduced 2004:
Modern Math Education with CAS, DGS and TC beginning in the 8th class upto 12th class, using graphic calculators (GTR)
CAS - Computer Algebra Systems
DGS - Dynamic Geometry Software
TC - Spreadsheet (Table Calculation)
GTR – Grafiktaschenrechner: Graphic Calculator

8th class:

- Knowledge of the use of CAS when forming more complex terms and equations
- Investigating the influence of parameters in the function equation to trace the graph with DGS, TC, GTR or CAS
- Finding equations for measurement series with the help of linear regression with with GTR, CAS or TC
- Solving linear systems of equations with more complex coefficients with GTR or CAS (two equations with two unknown variables)

9th class:

- Functions and Powers
- Mastered of determining zero quadratic functions, graphical solving quadratic equations and solving with GTR or CAS

10th class:

- Obtaining the inverse function with CAS, graphical interpretation
- Use of CAS to demonstrate the properties of functions
- Obtaining illustrative of the limit concept
- Know of parametric representation and polar coordinates to describe curves with GTR and CAS

11/12th class:

Differential calculus

- The use of CAS in particular, should promote discovery learning, and support for substantive tasks, the reflection on the facts and the interpretation of the result.

Integral Calculus

- The use of CAS in particular, should promote discovery learning, and support for substantive tasks, the reflection on the facts and the interpretation of the result.

worksheets:
- The use of worksheets in mathematics instruction has a long tradition.
- The use of a worksheet should guide the students to a structured work.
- Instead of an oral instruction, which requires a synchronous work of all students, the worksheet individually and with their own timing can be processed next.
- The sequence of the work orders in the worksheet helps to recognize the logical structure of a problem; the work procedures help to penetrate the question.

The disadvantage of a sheet of paper with work orders is seen, that the tools which can be used must be made available about. The students do not know always, how to carry out its solution steps in detail. Moreover, in a classic worksheet are missing the self check of the results, a feedback of the partial steps and also the visualization of the results.

The new developed **eActivity in the ClassPad** represents an extremely rich extension of the worksheet. The eActivity com-bines the written representation of the setting of tasks of a worksheet with the tool level of the ClassPad.

These tools are the individual menus or modules, which the ClassPad offers: Computer algebra system, dynamic geometry software, computer statistics, curve plotter, and much more.

eActivity:
- Thus, all tasks can be worked on with the possibilities of the computer.
- At the same time, the documentation of the work can be entered directly.
- The found results can to be visualized immediately or the results may be in a hidden file, can be viewed.
- It is an interactive work of the students, between setting of tasks and the results and control of the results themselves.

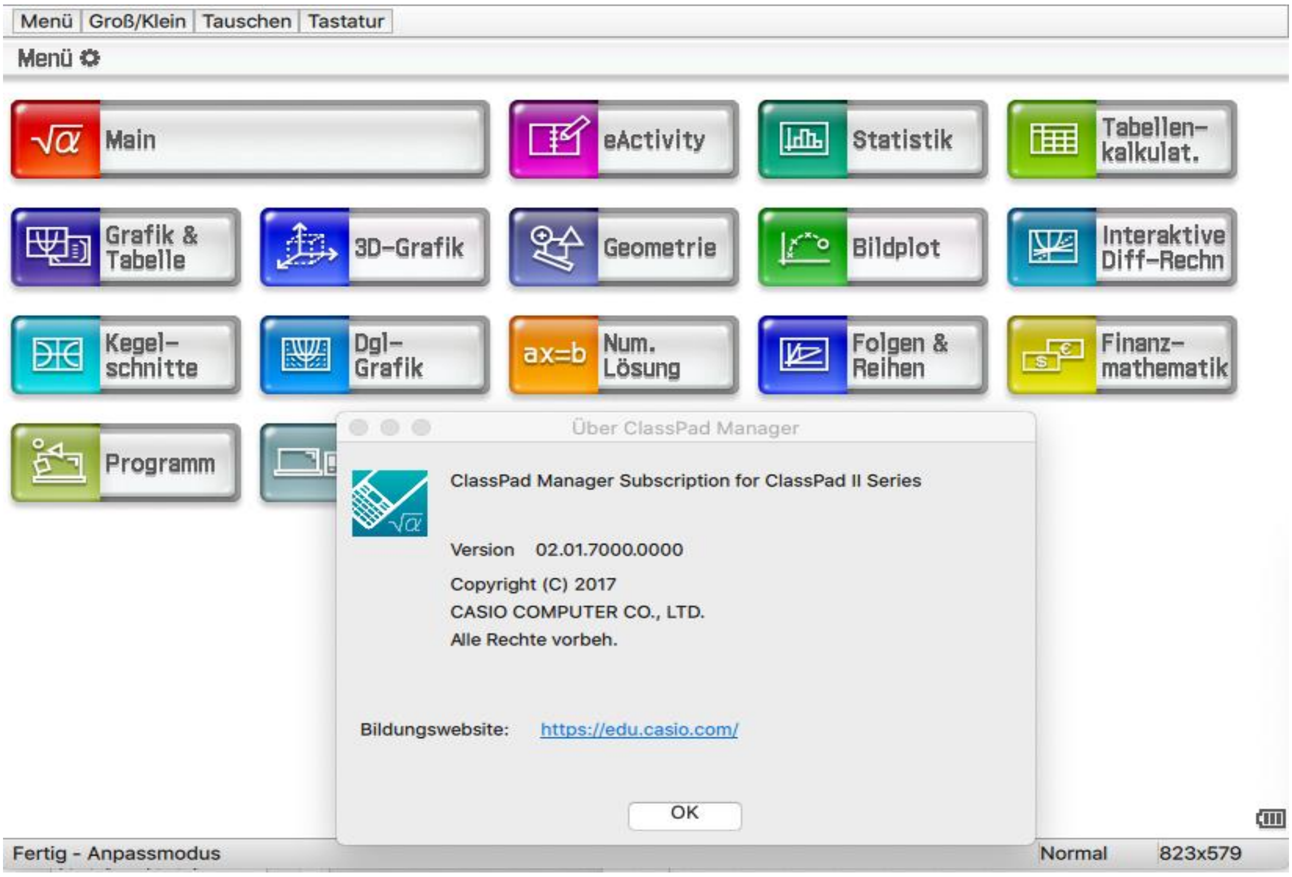
The tool – ClassPad 400: *http://edu.casio.ru/fx-cp400/* (Accessed Sept. 23, 2021)
- Graphic calculator with a touch screen
- The Fx-CP400 has a number of features to help students to learn better
- USB support for fast and easy data transfer and compatibility with **a CASIO projector** for displaying information on a whiteboard.

The tool – Beamer XJ-A146: *http://casio-projectors.ru/products/xja146/* (Accessed Sept. 23, 2021)

Picture 1. Graphic calculator with a touch screen and a CASIO projector

eActivity – an example

Differential Evolution in School Mathematics - a derivative-free method for calculating extreme values



Picture 2. Current software (September 2021)

Using a graphic method (3D graphics) and later the derivative-free search method (differential evolution): estimation of the parameters of the nonlinear regression function (binary logistic regression).

Setting up the problem:

Here the binary logistic regression is concerned with testing the effect of a variable **x** on a binary outcome **y**. To demonstrate the process of the regression, the discussion focuses on a question of obvious importance:

How does the volume **x** of an alarm clock affect the getting out of bed in the morning? Depending on the volume **x** (x=0 "very quiet" to x=5 "very loud") the ringing of an alarm clock leads to the fact that one "continues to sleep" (category y=0) or "wakes up and get up"(category y=1).

A set of random data is available:

It may happen that the quieter call is already heard, but the louder one is not heard. The example is suitable for school mathematics and shows the connection with the theory of probability.

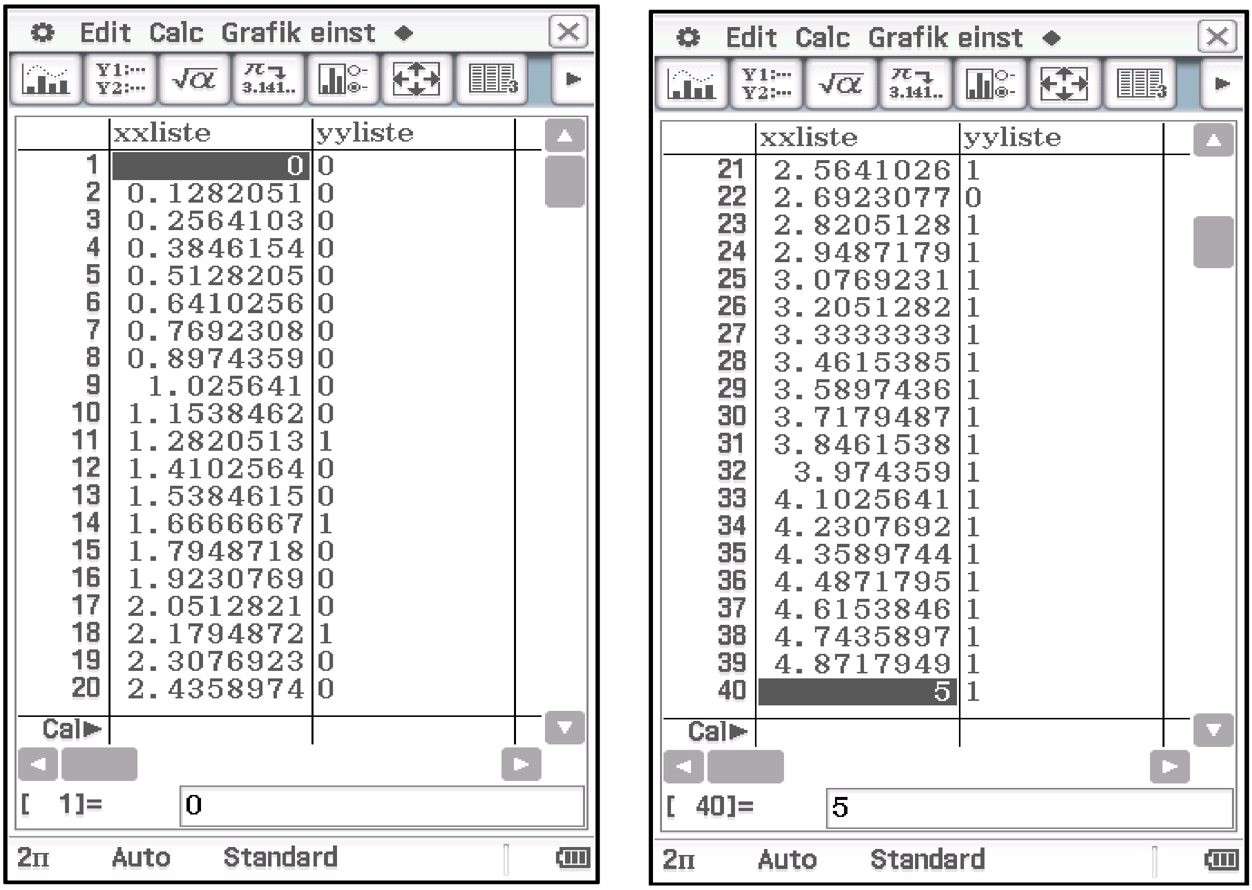
Here we examine the data material from the book by M.Scarpino [3] for n = 40 consecutive days with increasing alarm volume from x=0 to x=5 and the detected binary output y=y (x).

For the growing from 0 to 5 alarm volume x1 to x40 we have:

x1=0, x2=x1+5/39, x3= x2+5/39, …, x40=x39+5/39, that volume in data list xxliste.

The categories (detected binary output) y1 to y40 (0 or 1) are in the data list yyliste:

{0,0,0,0,0,0,0,0,0,0,1,0,0,1,0,0,0,1,0,0,1,0,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1}



	xxliste	yyliste
1	0	0
2	0.1282051	0
3	0.2564103	0
4	0.3846154	0
5	0.5128205	0
6	0.6410256	0
7	0.7692308	0
8	0.8974359	0
9	1.025641	0
10	1.1538462	0
11	1.2820513	1
12	1.4102564	0
13	1.5384615	0
14	1.6666667	1
15	1.7948718	0
16	1.9230769	0
17	2.0512821	0
18	2.1794872	1
19	2.3076923	0
20	2.4358974	0

	xxliste	yyliste
21	2.5641026	1
22	2.6923077	0
23	2.8205128	1
24	2.9487179	1
25	3.0769231	1
26	3.2051282	1
27	3.3333333	1
28	3.4615385	1
29	3.5897436	1
30	3.7179487	1
31	3.8461538	1
32	3.974359	1
33	4.1025641	1
34	4.2307692	1
35	4.3589744	1
36	4.4871795	1
37	4.6153846	1
38	4.7435897	1
39	4.8717949	1
40	5	1

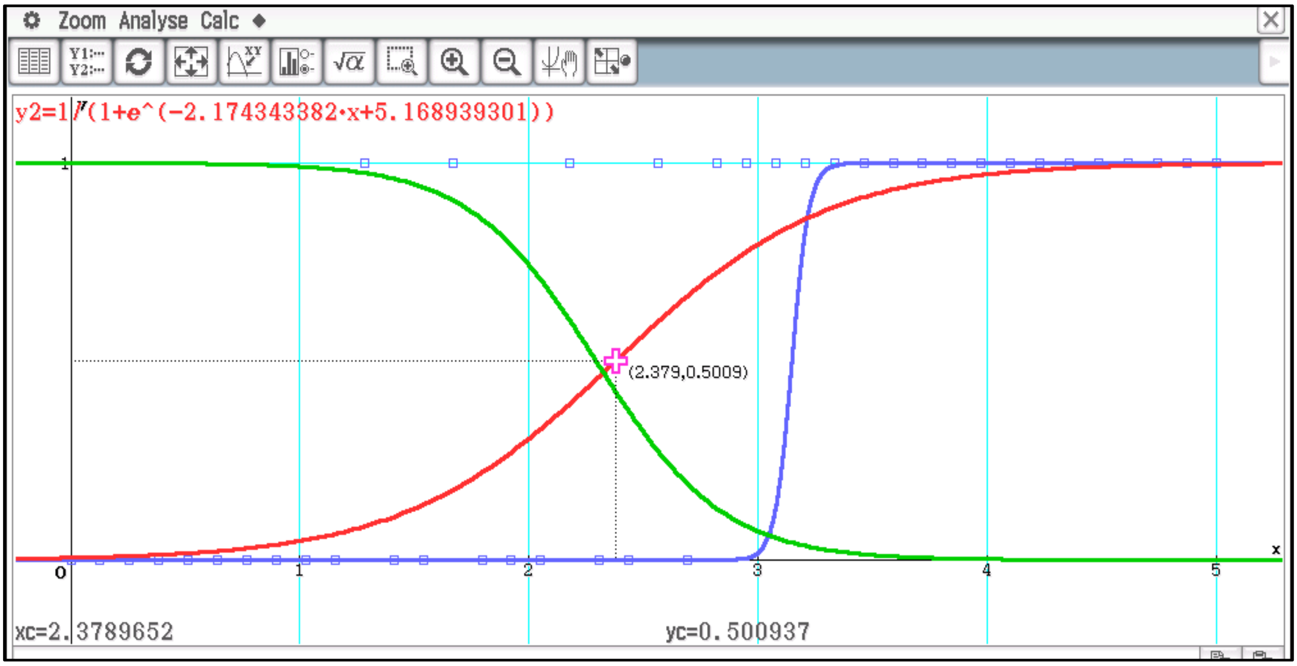
Picture 3.

For data processing (see Picture 3), the school mathematics software from CASIO is used: ClassPad-Manager-Subscription Version for ClassPad II Series Version 02.01.7000
see: (Accessed Sept. 23, 2021)
https://edu.casio.com/products/classroom/cp400/
https://support.casio.com/ru/support/osdevicePage.php?cid=004002065
https://casio-calcs.ru/products/fx-cp400/
http://edu.casio.ru/classpad2-fx-cp400-pri-obuchenii/

Defining the nonlinear regression model with the logistic function. The binary logistical regression is based on the following model approach:

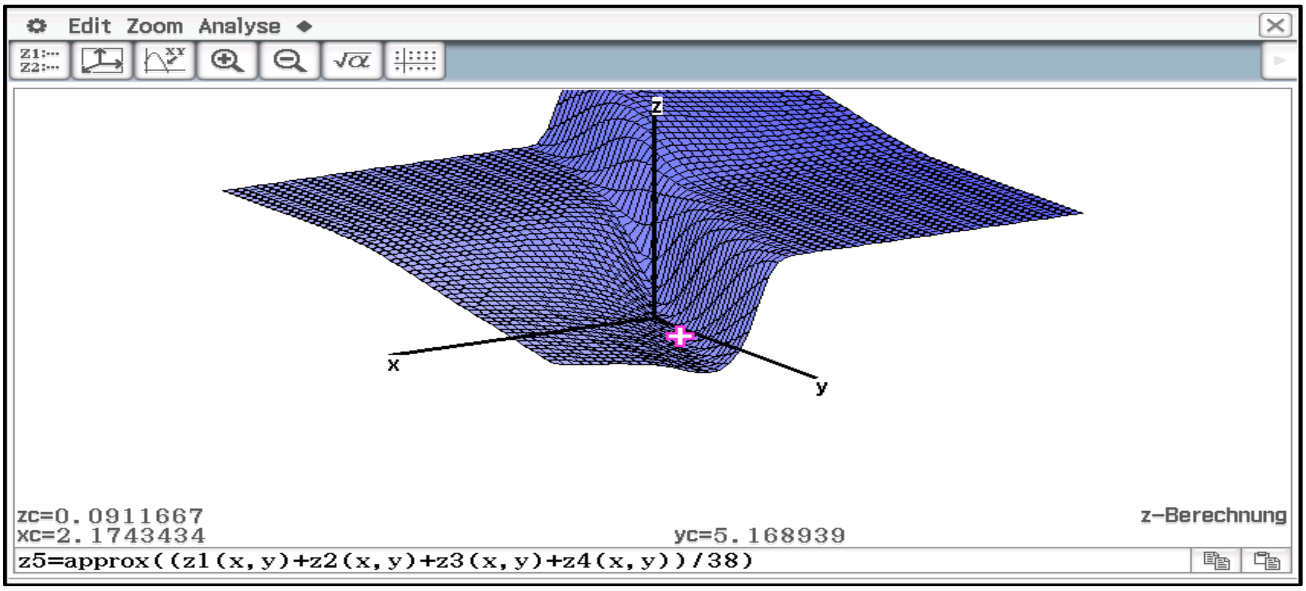
y(x)=1/(1+e^(-a*x+b)), a>0, b>0.

In Picture 4 we can see the 40 data points and various logistic functions:
e.g . the **blue curve**: y(x) = 1/(1+e^(-27*x+85)) is not optimal (the left 1 values are not included), the **green non-growing curve** is obviously not suitable: a = -3.6 < 0, b = -8.3 < 0



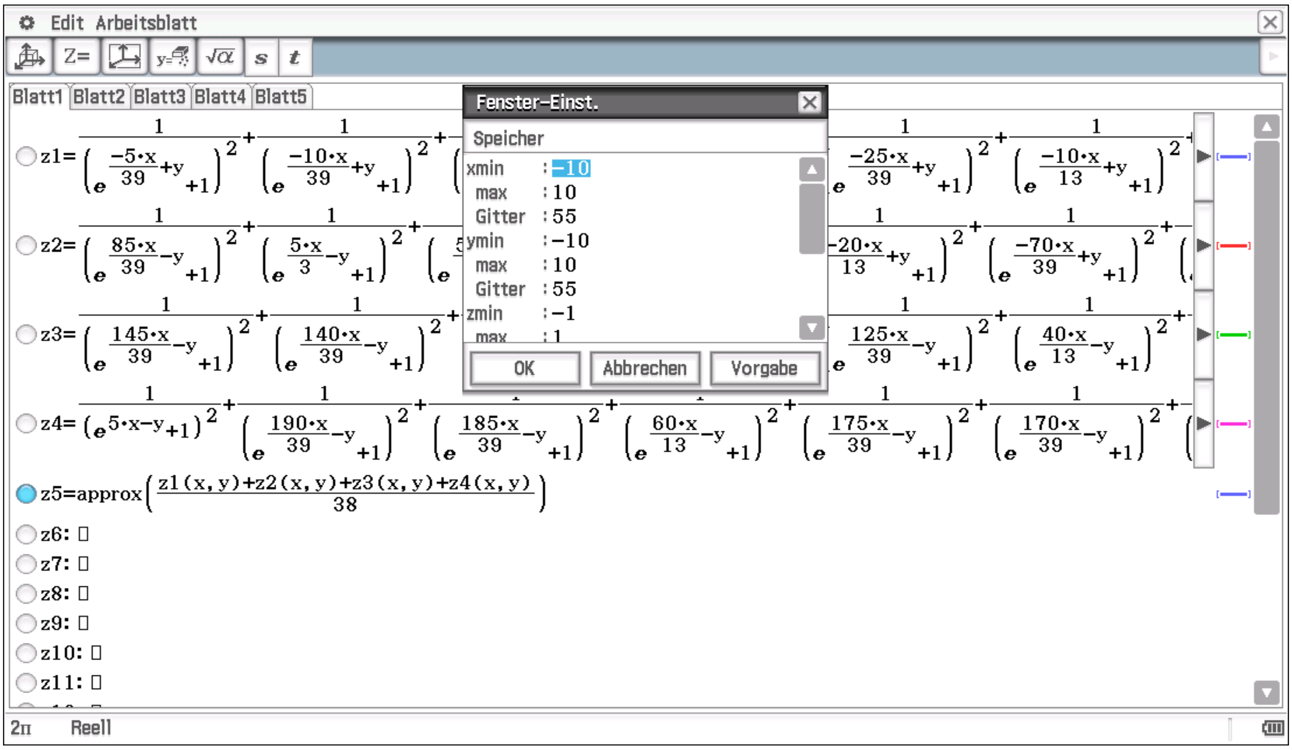
Picture 4.

The objective function is the MKQ function (MKQ method of the smallest squares):
MKQ(a,b)=1/(n-2) * sum((yi-y(xi))^2,i,1,n) → min!
The optimal solution is the **red curve** with a=2.17434, b=5.16894 (result with ClassPad software)
MKQ = 0.0911667
From the point of view of **probability calculation**, the **red curve** is a **distribution function**.
Examine the 3D graphics of the MKQ function to solve the min-problem:
MKQ(a,b)=1/(n-2) * sum((yi-y(xi))^2,i,1,n) → min! with y(x)=1/(1+e^(-a*x+b)).
The school software requires the standard variables x, y, z for the 3D graphics.
Hence we define z(x,y)=MKQ(x,y) with x=a and y=b. (Viewing cuboid: -10 < x <10, -10 < y <10, -1 < z <1, grid lines 55 in the x- and y-direction) (see Picture 5).

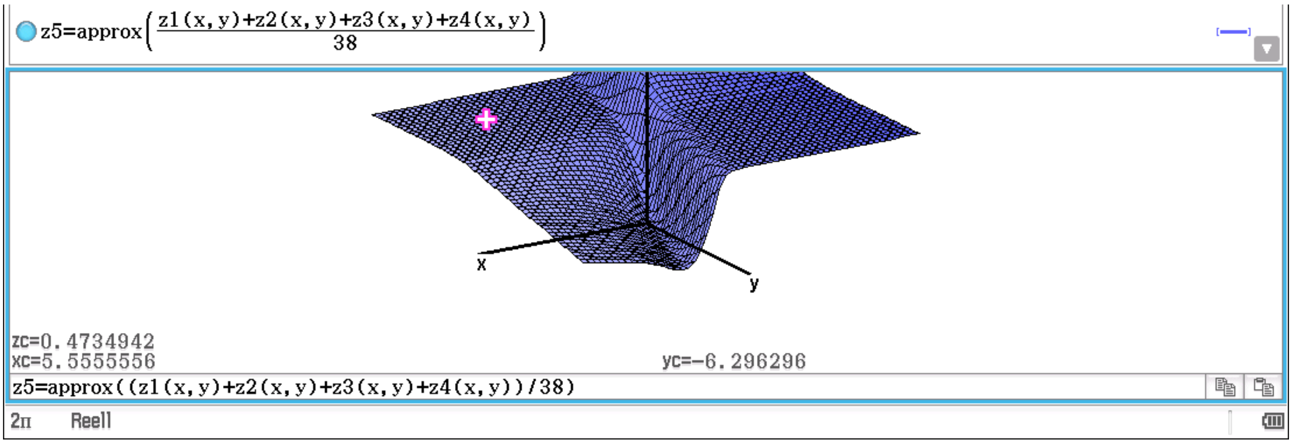


Picture 5.

Inputing of 40 MKQ terms for the 3D graphical display, we finally obtain:



Picture 6.



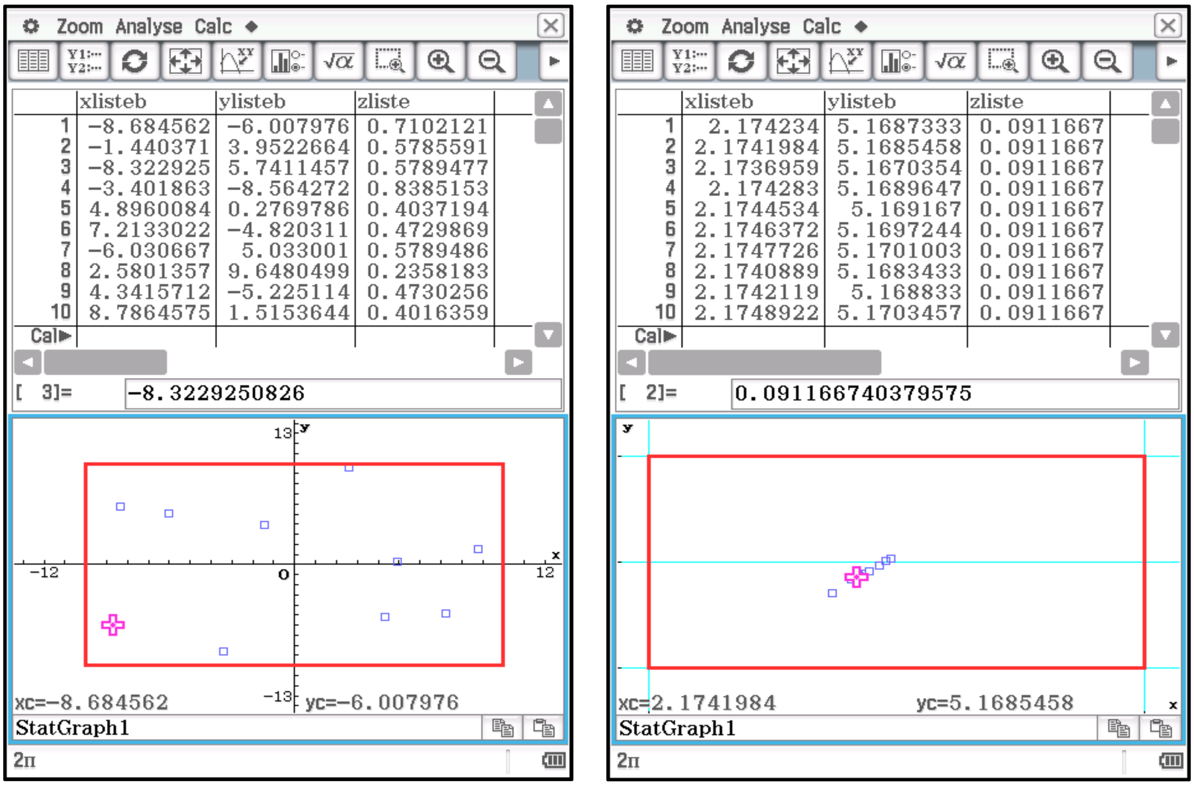
Picture 7. View in angle: theta = 58º, phi = 72º

Another way to solve the min-problem is optimization of the objective function z(x,y) by the method without derivatives: **differential evolution**, self-programming for ClassPad II.

The starting points are 10 random points in the definition area -10 < x <10, -10 < y <10. After 50 iteration steps, the 10 random points are all near the minimum position (see Picture 8):

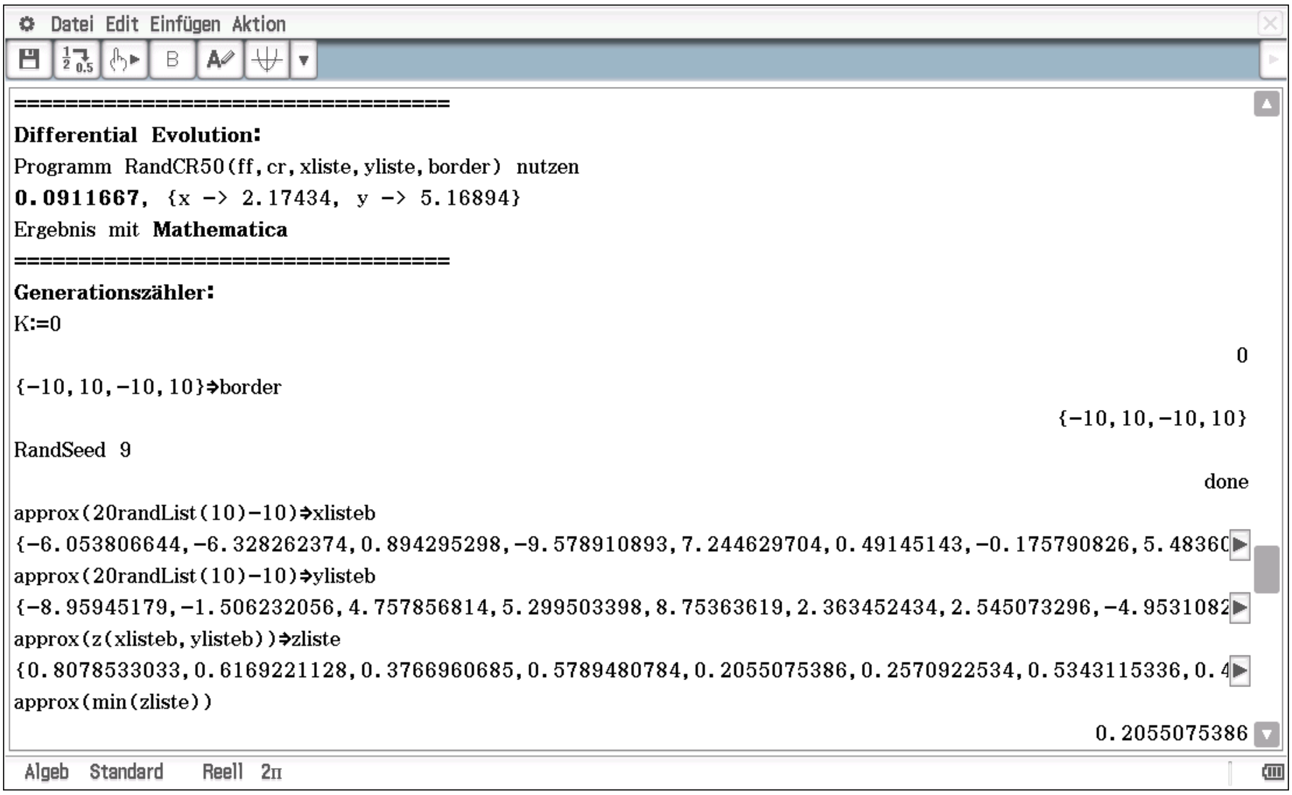
2.17 < x < 2.18 and 5.16 < y < 5.18.

The stochastic search method works derivative-free and uses the elementary vector calculation.



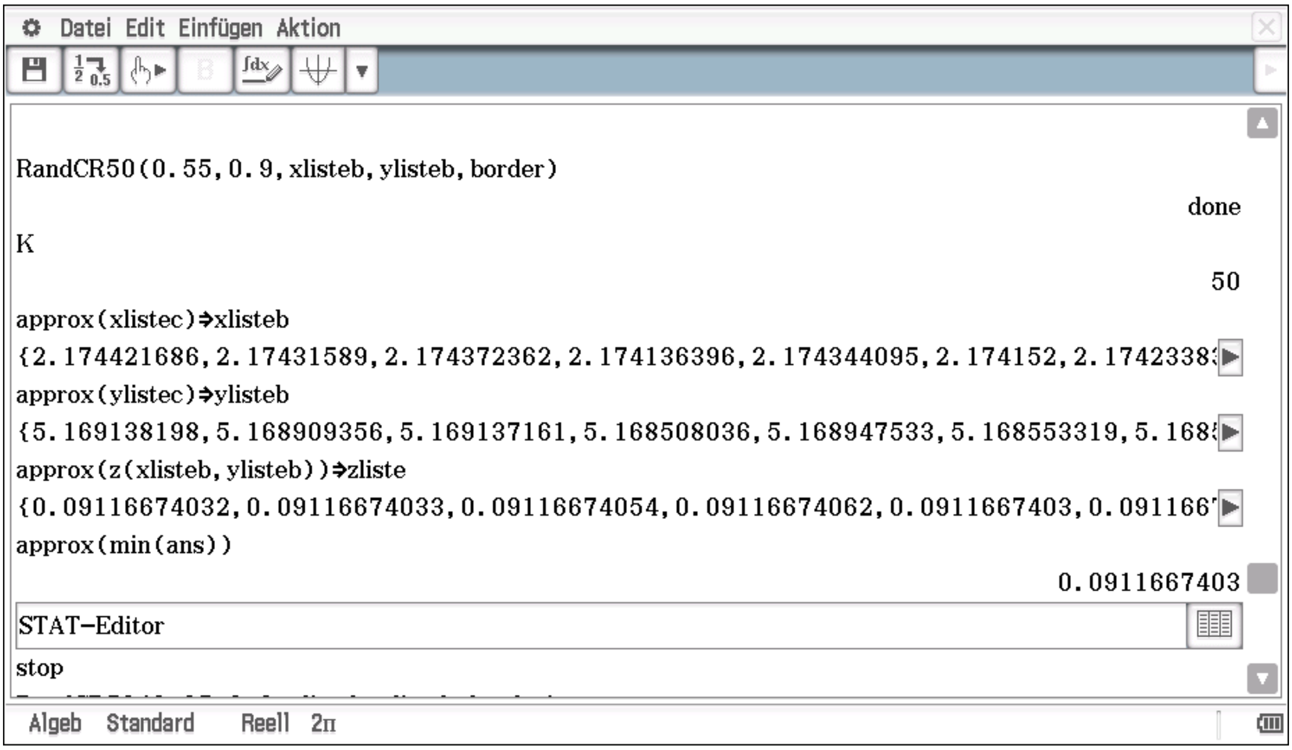
Picture 8.

Create 10 random starting points in the search area: using **randList**(10)



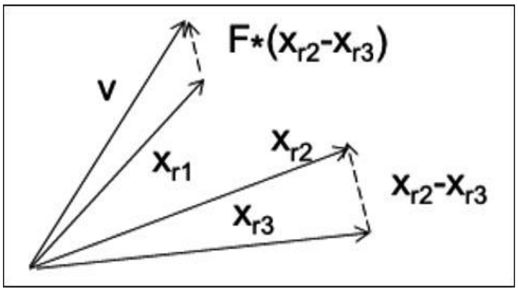
Picture 9.

Calling the program **RandCR50**(0.55,0.9, xlistb, ylistb, border):



Picture 10.

To the procedure:
For every vector **x** of the old generation, there is additionally three vectors from the old generation (**x**r1, **x**r2, **x**r3), determine the donor vector (**v**) as a linear combination of **x**r1, **x**r2, **x**r3:
v=**x**r1+F*(**x**r2-**x**r3), 0<F<2 (here F = 0.55, **mutation factor**).



Picture 11.

x and **v** together form the parent couple for the **recombination**. Create a trial vector (**u**) by mixing the elements of **x** and **v.** The mixture of the elements of **x** and **v** takes place random-controlled. **x**, **v**, **u** are vectors of the dimension D=2. CR is the **cross-over-rate**: 0<CR<1 (here CR=0.9).
j is now an integer random number (j=1 or j=2) and ri is a real random number: 0<ri<1.
j ensures that **x** and **u** differ in at least one element:
for i=1,2 now ui=vi, if ri<CR or i=j, otherwise ui=xi.
x and **u** form competitors in the **selection**.

Now select one of the two vectors **x**, **u** for the new generation. The selection is based exclusively on the basis of the quality (fitness) of an individual (vector). Only the better of the two individuals is transferred to the new generation.

The selection takes place deterministic, not random dependent:

x*=**u**, if MKQ(**u**)<MKQ(**x**), otherwise **x***=**x**.

MKQ is the quality function to be optimized (fitness function). If the quality is the same, the individual **x*** created by mutation and recombination is taken over into the new generation of 10 points (vectors). Here we created 50 generations to get the optimal solution, which improves the result by M. Scarpino [3].

Appendix.

The source code of the program **RandCR50** with the subprogram **RandCR02**:

```
'RandCR50(ff,cr,xliste,yliste,border)

Local zmini,zmini1
ClrText
approx(xliste)⇒xlisteb
approx(yliste)⇒ylisteb

approx(approx(min(z(xliste,yliste))))⇒zmini
Local L

For 1⇒L To 50 Step 1
  RandCR02(ff,cr,xlisteb,ylisteb,border)
  xlistec⇒xlisteb
  ylistec⇒ylisteb
  approx(min(approx(z(xlistec,ylistec))))⇒zmini1
  print "min neue Generation": print {K,approx(zmini1)}
Next


print "==============="
print "min alte Generation":print approx(zmini)
print "min neue Generation":print approx(zmini1)

return


'Prof.Dr.Ludwig Paditz 21.01.2020, korr.am 21.03.2021
'Differential Evolution mit Test -Koordinaten im Bereich-
'Programm RandCR02(ff,cr,xliste,yliste,border)
'Bereitstellung der Zufallsauswahl in randL und vekL [k,m₁,m₂,m₃,CR,CR]
'AD(=20) ... Dimension der zweidim. Punktegeneration
'sed ... zufälliger Startpunkt des Zufallsgenerators
'F(=0.8) Gewichtsfaktor für Vektordifferenz in v
'CR(=0.5) Vergleichswert für zufälliges Croosover
'border ... {xu,xo,yu,yo} Bereichsgrenzen

Local F,CR,AD,sed,xlistea,ylistea,randL,I,vekvx,vekux,vekvy,vekuy,veku,zxy1,zxy2,
zxy,zu
Local vekL,xu,xo,yu,yo,randL1,randL2,randL3,randL4,randL5,randL6,vekvxx,vekvyy
'Generationszähler
K+1⇒K

approx(ff)⇒F
approx(cr)⇒CR
dim(xliste)⇒AD
randList(1,1,10)-1⇒sed
RandSeed sed[1]
approx(border[1])⇒xu
approx(border[2])⇒xo
approx(border[3])⇒yu
approx(border[4])⇒yo


'ClrText

'Anfangslisten (Anfangsgeneration) xlistea,ylistea
approx(xliste)⇒xlistea: approx(xliste)⇒xlistec
approx(yliste)⇒ylistea: approx(yliste)⇒ylistec

'Generieren der benötigten Zufallsdaten (unabhängig von Datenpunkten und Zielfunktion)
'================================
'der aktuelle Index I(=1) wird ausgeschlossen,
'drei weitere unterscheidbare Indizes (m₁,m₂,m₃) per Zufallsauswahl ermitteln für Dona
tor-Vektor
'Zufallsindex (1 oder 2) für Koordinatenauswahl zum Crossover ermitteln
'für jede Koordinate zufälligen Vergleichswert für CR generieren

randList(AD,1,2)⇒randL1
randList(AD)⇒randL5
randList(AD)⇒randL6
randL1⇒randL2: randL1⇒randL3: randL1⇒randL4

For 1⇒I To AD Step 1
Lbl AA
 randList(3,1,AD)⇒randL
 If prod(randL-I)*(randL[1]-randL[2])*(randL[1]-randL[3])*(randL[2]-randL[3])
=0
 Then
```

```
 Goto AA
 IfEnd
randL[1]→randL2[I]
randL[2]→randL3[I]
randL[3]→randL4[I]
next

augment(augment(augment(augment(augment(listToMat(randL1),listToMat(randL2)),listTo
Mat(randL3)),listToMat(randL4)),listToMat(randL5)),listToMat(randL6))→vekL
approx(trn(vekL))→vekL


'Matrix vekL komplett generiert (6 Zeilen, AD Spalten)
'print approx(vekL)
'stop
'================================

'Erzeugung des Vektors v und aus zufallsgesteuerter Mischung von v und x ergibt sich u
'====================

seq(approx(xlistea[vekL[2,I]]+F*(xlistea[vekL[3,I]]-xlistea[vekL[4,I]])),I,1,AD,1)→
vekvx
seq(approx(ylistea[vekL[2,I]]+F*(ylistea[vekL[3,I]]-ylistea[vekL[4,I]])),I,1,AD,1)→
vekvy

For 1→I To AD Step 1
' Test, dass Koordinate vekvx im vorgegebenen Gebiet bleibt
 If approx((vekvx[I]-xu)*(vekvx[I]-xo))>0
  Then
  randList(1)*(xo-xu)+xu→vekvxx
  vekvxx[1]→vekvx[I]
 IfEnd
' Test, dass Koordinate vekvy im vorgegebenen Gebiet bleibt
 If approx((vekvy[I]-yu)*(vekvy[I]-yo))>0
  Then
  randList(1)*(yo-yu)+yu→vekvyy
  vekvyy[1]→vekvy[I]
 IfEnd
Next

vekvx→vekux
vekvy→vekuy

For 1→I To AD Step 1
 If vekL[5,I]*(vekL[1,I]-1)>CR
  Then
  xlistea[I]→vekux[I]
 IfEnd
 If vekL[6,I]*(2-vekL[1,I])>CR


 Then
 ylistea[I]→vekuy[I]
 IfEnd
Next

approx(trn(augment(listToMat(vekvx),listToMat(vekvy))))→veku

'Auswahl des besseren Vektors x oder u mittels Zielfunktion (Gütefunktion, Fitnessfunktion)
'Ergebnislisten (neue Generation) xlistec,ylistec

For 1→I To AD Step 1
 approx(z(xlistea[I],ylistea[I]))→zxy
 approx(z(veku[1,I],veku[2,I]))→zu
 If approx(zu)≤approx(zxy)
 Then
 veku[1,I]→xlistec[I]
 veku[2,I]→ylistec[I]
 Else
 xlistea[I]→xlistec[I]
 ylistea[I]→ylistec[I]
 IfEnd

Next
return
```

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ДИФФЕРЕНЦИАЛЬНАЯ ЭВОЛЮЦИЯ В ШКОЛЬНОЙ МАТЕМАТИКЕ – МЕТОД ВЫЧИСЛЕНИЯ ЭКСТРЕМАЛЬНЫХ ЗНАЧЕНИЙ БЕЗ ПРОИЗВОДНЫХ

Л. Падитц

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С помощью метода поиска без производных (дифференциальная эволюция) выполняется оценка параметров функции нелинейной регрессии (двоичная логистическая регрессия). Доступны случайные данные: в зависимости от громкости сигнала будильника x (x=0 «очень тихо» до x=5 «очень громко») звонок будильника приводит к тому, что человек «продолжает спать» (категория y=0) или «просыпается и встает» (категория y=1). Может случиться так, что более тихий звонок уже слышен, а более громкий не слышен. Пример подходит для школьной математики и показывает связь с теорией вероятностей.

Ключевые слова: *Связь школьной математики и теории вероятностей, метод поиска без производных, дифференциальная эволюция, оценка параметров, двоичная логистическая регрессия, нелинейная регрессия, Метод наименьших квадратов, расчет вектора.*

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